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Some New Multi-Symmetric Packings of Equal Circles on a 2-Sphere

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Abstract

In an earlier paper [Tarnai & Gáspár (1987). *Acta Cryst.* A**43**, 612–616] four packing sequences were investigated to find locally extremal arrangements of equal circles on a sphere in tetrahedral, octahedral and icosahedral symmetry. In this paper one of these sequences is continued to find relatively dense arrangements in the case of large numbers of circles (n = 150, 216, 300, 432, 750, 1080).

Introduction

The Tammes problem is as follows: how must n equal non-overlapping circles be packed on a sphere so that the angular diameter of the circles will be as great as possible? With the analogy of the capsid structure of small 'spherical' viruses, Tarnai & Gáspár (1987) constructed some locally extremal arrangements in tetrahedral, octahedral and icosahedral symmetry. Regular tessellations were considered on these regular polyhedra. After Coxeter (1972) these tessellations were denoted by the symbol $\{3,q+\}_{b,c}$, where the number 3 means that the tessellation consists of equilateral triangles and the notation $q+$ refers to the fact that q or more than q triangles meet at the vertices of the tessellation. The suffixes b, c denote the coordination numbers of triangulation.

Four sequences of circle packings were investigated for $q = 3$, 4, 5: the tessellations $\{3,q+\}_{c+1,c}$ and $\{3,q+\}_{c+2,c}$ by removal and preservation of the vertices of the regular polyhedra $\{3,q\}$. For the calculations a procedure has been developed based on the 'heating technique' (Tarnai & Gáspár, 1983) considering the graph as a spherical bar and joint structure.

The first three (c = 1, 2, 3) elements of the four sequences were calculated by Tarnai & Gáspár (1987). The second sequence, the tessellation $\{3,q+\}_{c+2,c}$ with removal of the vertices of the base polyhedron, showed a strong regularity: for every c the topology of the subgraph on a face of the regular polyhedron was the same in the case of the tetrahedron, octahedron and icosahedron, and all of the subgraphs consist only of quadrangles with the exception of the middle of the face and the neighbourhood of the vertices where there are triangles. This regularity led us to continue this sequence to find relatively dense packings for large numbers of circles.

New results

In the case of $c = 4$ the regularity of the packing remained. The subgraph of these packings can be seen in Fig. 1 in a schematic form where (and similarly in Fig. 2) each great equilateral triangle composed of dashed lines is a face of the regular tetrahedron or

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Table 1. *Close packings of congruent spheres on a spherical surface*

n	Tessellation	Subgraph	Diameter d (°)	Angles in the graph (°) α	β	γ	δ	Density
150*	$\{3,3+\}_{6,4}$	Fig. 1	17·028598	97·58380	109·81780	97·58380		0·82658
216	$\{3,3+\}_{7,5}$	Fig. 2(*a*)	14·211838	98·74436	115·84175			0·82952
300	$\{3,4+\}_{6,4}$	Fig. 1	12·006657	78·74622	84·70472	78·46819		0·82263
432	$\{3,4+\}_{7,5}$	Fig. 2(*b*)	9·983440	80·09028	88·34216	89·10196	78·02241	0·81922
750	$\{3,5+\}_{6,4}$	Fig. 1	7·746738	67·39066	69·76133	67·31882		0·85657
1080	$\{3,5+\}_{7,5}$	Fig. 2(*b*)	6·446068	67·07863	71·36495	71·50135	67·29658	0·85417

* The packing in $\{3,4+\}_{4,3}$ is better (Tarnai & Gáspár, 1987).

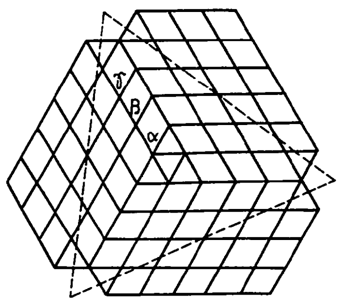
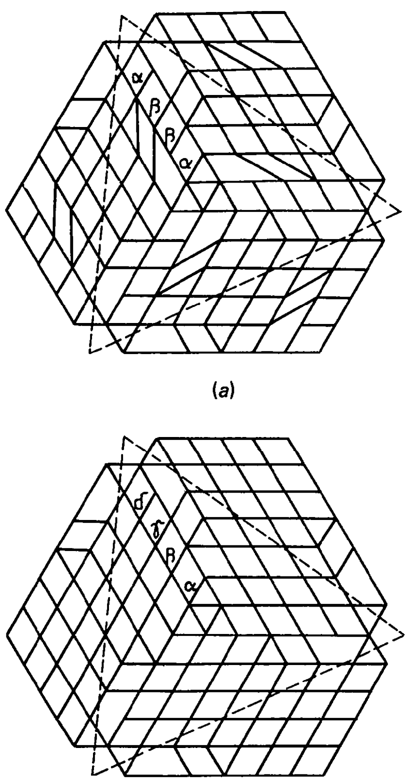


Fig. 1. Subgraph of the packing in system $\{3,q+\}_{6,4}$ with removal of the vertices of the base polyhedron for $q = 3, 4, 5$.



(*a*)

(*b*)

Fig. 2. Subgraph of the packing in system $\{3,q+\}_{7,5}$ with removal of the vertices of the base polyhedron: (*a*) $q = 3$; (*b*) $q = 4, 5$.

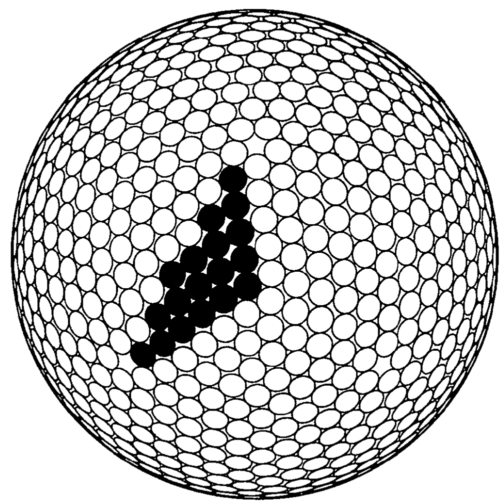


Fig. 3. Packing of 1080 circles on a sphere (drawn by G. Koller).

octahedron or icosahedron, so packings for $n = 150$, 300, 750 were obtained. The packing of 150 circles in $\{3,4+\}_{4,3}$ (Tarnai & Gáspár, 1987) was better than the present one. In the case of $c = 5$ the regularity ceased to exist. The subgraph is shown in Fig. 2(*a*) in the case of tetrahedral symmetry ($n = 216$), and the common subgraph of packings of octahedral and icosahedral symmetry ($n = 432$, 1080) is shown in Fig. 2(*b*).

Table 1 shows the data for these six arrangements, together with some angles to allow the graphs to be reproduced easily. Günther Koller (Trollhättan, Sweden) has produced computer graphics of these arrangements. The case of $n = 1080$ is shown in Fig. 3, where 18 circles (a third of a face) are darkened. All the other arrangements can be obtained by icosahedral symmetry.

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